between the two values.

Method of Calculating the Median :

Individual Series : To calculate median in individual series, the following process has to be followed :

(i) Arrange the data in ascending or descending order. (both arrangements would give the same answer).

(ii) Apply the following formula :

$$\text{Median } (M) = \text{Size of } \frac{N+1}{2} \textit{th item}$$

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where, N = Number of items.

If the number of items in the array is odd, we can easily find out the value of median. On the other hand, if the number of items in the array is even, the practice is to take the mean of the values of the middle items, as the median must lie between them. If the students of a class, 13 (odd) in number, are asked to stand in order of their heights, the 7th student from either side shall be the one whose height will be the median height of the class. If the students of a class, 14 (even) in number, are asked to stand in order of their heights, the size of $\left(\frac{N+1}{2}\right)$ th item will be falling in between 7th and 8th student i.e., 7·5th item. Then we have to take the average of the heights of the 7th student and the 8th student as the median height of the class.

***Illustration* 15.**

Find out the median-value of following data :

10, 9, 20, 36, 14, 8, 19, 32, 28, 16, 5, 60, 40

Solution.

First of all, we will arrange the given data in ascending order :

S. No. :	1	2	3	4	5	6	7	8	9	10	11	12	13
Item-Value :	5	8	9	10	14	16	19	20	28	32	36	40	60

$$M = \text{Size of } \left(\frac{N+1}{2}\right) th \text{ item} = \text{Size of } \left(\frac{13+1}{2}\right) = 7\text{th item}$$

Now, value of 7th item in above arranged data in ascending order i.e. 19 will be median.

$\therefore$ Median or M = 19

***Illustration* 16.**

In a batch of 15 students 5 were failed in a test. The marks of 10 students who passed were : 9, 6, 7, 8, 8, 9, 6, 5, 4, 7 what was the median of the marks of all the 15 students ?

Solution.

In the above question, marks of 10 students are given who passed but marks of 5 students who failed are not known. Unknown marks will be taken as A, B, C, D, E and items in ascending order will be as follows :

A, B, C, D, E, 4, 5, 6, 6, 7, 7, 8, 8, 9, 9,

Median marks of the students :

$$M = \text{Size of } \left(\frac{N+1}{2}\right) th \text{ student}$$

$$= \text{Size of } \left(\frac{15+1}{2}\right) th \text{ student}$$

$$= \text{Size of } 8\, th \text{ student} = 6$$

$\therefore$ Median = 6

***Illustration* 17.**

The marks obtained by 16 students of a class in statistics-test are given below. Find out the median-marks :

S. No. :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Marks :	5	12	17	23	28	31	37	41	42	49	54	58	65	68	73	77

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Solution.

$$\text{Median (M)} = \text{Size of } \left(\frac{N+1}{2}\right) th \text{ item}$$

$$= \text{Size of } \left(\frac{16+1}{2}\right) th \text{ item or Size of } 8 \cdot 5 \, th \text{ item}$$

$$\text{Where Size of } 8 \cdot 5\text{th item} = \frac{\text{Value of 8th item + Value of 9th item}}{2} = \frac{41+42}{2}$$

$$\therefore \quad \text{Median Marks} = \mathbf{41 \cdot 5}$$

Note : Marks obtained are already arranged in ascending order, hence there is no need to arrange them in ascending order.

Discrete Series : (i) Arrange the data in ascending or descending order of magnitude.

(ii) Find the cumulative frequencies.

(iii) Apply the following formula :

$$\text{Median (M)} = \text{size of } \left(\frac{N+1}{2}\right) \text{th item}$$

(iv) Now look at the cumulative frequency column. The cumulative frequency in which the median size lies, its corresponding value will be median.

***Illustration* 18.**

Find Median of the following series :

Size	:	1	2	3	4	5	6	7	8	9	10
Frequency	:	2	7	17	29	38	41	40	30	17	6

Solution.

Location of Median

X	f	c.f.
1	2	2
2	7	9
3	17	26
4	29	55
5	38	93
6	41	134
7	40	174
8	30	204
9	17	221
10	6	227
$N = \Sigma f = 227$		

$$M = \text{Size of } \left(\frac{N+1}{2}\right) th \text{ item}$$

$$\text{or} = \text{Size of } \left(\frac{227+1}{2}\right) = 114th \text{ item}$$

It is clear from the cumulative frequency column that median size item 114 lies in 134 C.F. and its corresponding size 6 is median.

$$\therefore \quad M = 6$$

Zero Frequency in Discrete Series : If in discrete series, frequencies of one or more items are zero and on account of this median number is lying in two or more cumulative frequencies, the value (size) of such first cumulative frequency would be median.

***Illustration* 19.**

Calculate the value of the median in the following series :

x	:	2	5	6	8	10
f	:	6	10	0	0	14

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Solution.

X	f	c.f.
2	6	6
5	10	16
6	0	16
8	0	16
10	14	30

$$\text{Median} = \text{Size of } \left(\frac{N+1}{2}\right)th \text{ item}$$

$$= \text{Size of } \left(\frac{30+1}{2}\right)th \text{ item}$$

$$= 15{\cdot}5\,th \text{ item}$$

Middle item 15·5 lies in three cumulative frequencies. The value corresponding to first such cumulative frequency is 5. So, median = 5

Middle item as somewhere between succeeding number to any cumulative frequency in discrete series : If middle item is in the form of such fraction, which is somewhere between succeeding number to any cumulative frequency, the value of median will be calculated by dividing the sum of the item of such cumulative frequency and the next item by two as elaborated in the following example.

***Illustration* 20.**

Find out Median from the following data :

Marks in Test :	6	7	8	9	10	11	12	13	14
No. of Students :	10	5	2	2	2	3	4	4	2

Solution.

X	f	c.f.
6	10	10
7	5	15
8	2	17
9	2	19
10	2	21
11	3	24
12	4	28
13	4	32
14	2	34

$$\text{Median} = \text{Size of } \left(\frac{N+1}{2}\right)th \text{ item}$$

$$= \text{Size of } \left(\frac{34+1}{2}\right)th \text{ item}$$

$$= 17{\cdot}5\,th \text{ item}$$

17·5th item lies in the succeeding number to cumulative frequency 17. So

$$\text{Median} = \frac{8+9}{2} = 8{\cdot}5$$

Explanation : In this illustration, there are 17 items related to values from 6 to 8 and from 9 to 14. Median is the value, which divides the series into two equal parts. So 8.5 is the value, which divides 17 items on each side.

Continuous Series : To calculate median in continuous series, the following process is to be followed :

(i) Find the cumulative frequencies.

(ii) Apply the following formula to get the middle item :

$$\text{Middle item } (m) = \frac{N}{2}th \text{ item}$$

(iii) On the basis of middle item, determine the particular class in which the value of median lies. The cumulative frequency in which the middle item lies, its corresponding class will be median class.

(iv) When the median class is located, the median value is to be interpolated with the help of the following formula :

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$$M = L_1 + \frac{L_2 - L_1}{f}(m - c) \text{ OR } M = L_2 - \frac{L_2 - L_1}{f}(m - c)$$

where, M = Median

L_1 = Lower limit of median class

L_2 = Upper limit of median class

f = Frequency of the median class

m = Middle item or median number

c = C.F. of the group just preceeding the median class.

Note : We can write 'i' instead of $L_2 - L_1$. 'i' means the class-interval of median class.

***Illustration* 21.**

Calculate median from the following :

Wages in Rs. :	30–40	40–50	50–60	60–70	70–80
No. of Workers :	50	54	85	45	30

Solution.

Calculation of Median

Wages	(f)	(c.f.)
30–40	50	50
40–50	54	104 c
50–60	85 f	189
60–70	45	234
70–80	30	264
	N = 264	

m = size of $\left(\frac{N}{2}\right)$ *th* item

$= \frac{264}{2} = 132$ *th* item

132th item lies in 189 cumulative frequency, therefore 50–60 will be median class.

$$M = L_1 + \frac{i}{f}(m - c)$$

Here, $L_1 = 50, i = 10, f = 85, m = 132, c = 104$

$$= 50 + \frac{10}{85}(132 - 104) \text{ or } = 50 + \frac{10 \times 28}{85} = 50 + 3{\cdot}29$$

$\therefore$ Median Wages = Rs. 53·29

Some Important Points About Median in Continuous Series

(1) Descending Class-intervals : Some times we see that given frequency distribution is in descending order instead of ascending order. To calculate median in such type of questions, there are following two ways :

(i) Convert the descending order into ascending order and in this way question becomes normal.

(ii) Solve the question in descending order by applying the following alternative formula :

$$M = L_2 - \frac{L_2 - L_1}{f}(m - c)$$

See the following illustration for clarification :

***Illustration* 22.**

Determine median of following data :

C.I. :	75–85	65–75	55–65	45–55	35–45	25–35	15–25
F :	3	6	8	14	3	2	2

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Solution.

First of all we will convert the given C.I. from descending to ascending order and then find out the cumulative frequency :

(C.I.)	(*f*)	(c.*f*.)
15–25	2	2
25–35	2	4
35–45	3	7 (c)
45–55	14 (*f*)	21
55–65	8	29
65–75	6	35
75–85	3	38
	N = 38	

$$m = \text{Size of } \frac{N}{2} th \text{ item} = \frac{38}{2} = \text{Size of 19th item}$$

19th item lies in 21 C.F. and represent 45–55 that is median class. Now we will use following formula to find out the value of median :

$$M = L_1 + \frac{L_2 - L_1}{F}(m - c)$$

$$= 45 + \frac{55 - 45}{14}(19 - 7)$$

or $$M = 45 + \frac{10 \times 12}{14}$$

$$= 45 + \frac{120}{14}$$

$$= 45 + 8 \cdot 57 = 53 \cdot 57 \text{ or } 53 \cdot 6$$

$\therefore$ Median = 53·6

If we want to solve the question in descending order then solution would be as follows :

(X)	(*f*)	(c.*f*.)
75–85	3	3
65–75	6	9
55–65	8	17 (c)
45–55	14 (*f*)	31
35–45	3	34
25–35	2	36
15–25	2	38
	N = 38	

$m = \frac{N}{2} th$ item $= \frac{38}{2} = 19th$ item which lies in 31 C.F. and represent 45–55 that is median class.

$$M = L_2 - \frac{L_2 - L_1}{F}(m - c)$$

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$$= 55 - \frac{55 - 45}{14}(19 - 17)$$

$$= 55 - \frac{20}{14}$$

$$= 55 - 1{\cdot}43$$

$$= 53{\cdot}57 \text{ or } 53{\cdot}6$$

$$\therefore \quad M = 53{\cdot}6$$

Hence, it is clear that both methods give the same answer.

Inclusive Class-intervals : If inclusive class-intervals have been given in the question then to find out the median, it is essential to convert the inclusive class-intervals into exclusive class-intervals.

***Ilustration* 23.**

Calculate median marks obtained from the following series :

Marks obtained :	0–9	10–19	20–29	30–39	40–49
No. of Students :	8	30	40	12	10

Solution.

To convert the above inclusive class-interval into exclusive class-interval, first C.I. 0–9 would be $-0{\cdot}5$ to $9{\cdot}5$. But marks obtained can never be negative. Therefore first exclusive C.I. will be $0–9{\cdot}5$ but class-interval from the point of view of median will be assumed 10.

Marks Obtained	No. of Students (f)	c. f.
$0–9{\cdot}5$	8	8
$9{\cdot}5–19{\cdot}5$	30	38
$19{\cdot}5–29{\cdot}5$	40	78
$29{\cdot}5–39{\cdot}5$	12	90
$39{\cdot}5–49{\cdot}5$	10	100

Median Group :

$$m = \text{Size of } \left(\frac{N}{2}\right) th \text{ term}$$

$$= \text{Size of } \left(\frac{100}{2}\right) th \text{ term}$$

$$m = \text{Size of } 50\, th \text{ item}$$

50th item lies in 78 C.F., hence median class will be $19{\cdot}5–29{\cdot}5$

$$M = L_1 + \frac{i}{f}(m - c)$$

$$= 19{\cdot}5 + \frac{10}{40}(50 - 38)$$

$$= 19{\cdot}5 + \frac{120}{40} = 19{\cdot}5 + 3$$

$$M = 22{\cdot}5 \text{ Marks obtained}$$

Note : When lower limit of first inclusive class-interval is zero and we want to convert the inclusive series into exclusive series, then normally we do not use negative value for lower limit of first exclusive class-interval. But there is no hard and

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fast rule to do so. In this regard we should analyse the nature of given data. If the given data are of such type which may be negative (for example temperature etc.) then lower limit of first exclusive C.I. should be negative.

Determination of median when mid value, mid point or central size is given : If mid value, mid point or central size has been given in the question then to find out the median, it is essential to construct the class-intervals.

When first class interval becomes median class : When first class of given frequency distribution is the median class then value of 'C' is to be assumed as zero and the remaining procedure will be same.

Determination of median when cumulative frequency distribution is given : If cumulative frequency distribution has been given in the question, then first of all we shall convert it into normal frequency distribution and after it total procedure will be the same.

See the following illustration for clarification :

***Illustration* 24.**

Find out median marks from the following frequency distribution :

Marks Less than :	80	70	60	50	40	30	20	10
No. of Students :	240	190	125	95	75	60	40	25

Solution.

Let us convert the data from less than frequency distribution into normal frequency distribution. Also convert the descending order into ascending order.

Marks	No. of Students	C.F.
0–10	25	25
10–20	15 = (40 – 25)	40
20–30	20 = (60 – 40)	60
30–40	15 = (75 – 60)	75
40–50	20 = (95 – 75)	95
50–60	30 = (125 – 95)	125
60–70	65 = (190 – 125)	190
70–80	50 = (240 – 190)	240
	N = 240	

m = Size of $\left(\frac{N}{2}\right)$ *th* item $= \frac{240}{2} = 120$ *th* item, which lies in 125 cumulative frequency and represent 50–60 as median class.

By applying the following formula :

$$M = L_1 + \frac{i}{f}\left(\frac{N}{2} - c\right) = 50 + \frac{10}{30}(120 - 95)$$

$$= 50 + \left(\frac{10}{30} \times 25\right) = 50 + \frac{250}{30} = 50 + 8{\cdot}33 = 58{\cdot}33$$

$\therefore$ $M = 58{\cdot}33$

Open-end class-intervals : Median is a positional average, therefore it is best average in case of open-end class-intervals. Hence, in such type of questions there is no need to complete the class to find out the value of median and there is also no change in formula or procedure. See the following illustration for clarification :

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***Illustration* 25.**

Find out the median from the following data :

Class :	–9	10–19	20–29	30–39	40–49	50–59	60–69	70–
Frequency :	22	38	54	75	72	64	31	10

Solution.

Since there is an inclusive class-interval distribution, we shall convert it into exclusive class-interval series.

Calculation of Median

C.I.	f	c. f.	Exclusive C.I.
–9	22	22	–9·5
10–19	38	60	9·5–19·5
20–29	54	114	19·5–29·5
30–39	75	189	29·5–39·5
40–49	72	261	39·5–49·5
50–59	64	325	49·5–59·5
60–69	31	356	59·5–69·5
70–	10	366	69·5–

Median

m = Size of $\left(\frac{N}{2}\right)th$ item $= \frac{366}{2} = 183th$ item which lies in 189 C.F. and represent 29·5–39·5 as median-class.

$$M = L_1 + \frac{i}{f}(m - c)$$

$L_1 = 29{\cdot}5$, i = 10, f = 75, c = 114

$$M = 29{\cdot}5 + \frac{10}{75}(183 - 114)$$

$$\text{or} = 29{\cdot}5 + \frac{10 \times 69}{75} = 29{\cdot}5 + 9{\cdot}2 = 38{\cdot}7$$

$\therefore$ $M = \mathbf{38{\cdot}7}$

Median in Unequal class-intervals : If unequal class-intervals have been given in the question, firstly convert them into equal class-intervals. After that procedure of determining the median will be as usual. If it is not possible to make the equal class-intervals, then we will calculate the median without making any change in the question. See Illustration 26 and 27 for clarification.

***Illustration* 26.**

Amend the following table and locate the median from the so amended table :

Size	Frequency
10–15	10
15–17·5	15
17·5–20	17
20–30	25
30–35	28
35–40	30
40 and above	40

Solution.

First of all, we will re-arrange the given frequency distribution on the basis of equal magnitude of 10.

Location of Median

Size	Frequency	c. f.
10–20	10 + 15 + 17 = 42	42
20–30	25	67
30–40	28 + 30 = 58	125
40–50	40	165
	N = 165	

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$$m = \text{size of } \left(\frac{N}{2}\right) th \text{ item} = \frac{165}{2} = \text{size of } 82 \cdot 5th \text{ item}$$

which lies in 30–40 class interval.

By interpolation : $M = L_1 + \frac{i}{f}(m - c)$

$$L_1 = 30, i = 10, f = 58, m = 82 \cdot 5$$

By substituting the values in the above formula :

$$M = 30 + \frac{40 - 30}{58}(82 \cdot 5 - 67)$$

$$M = 30 + \frac{10 \times 15 \cdot 5}{58} = 30 + 2 \cdot 67$$

$\therefore$ $M = 32 \cdot 67$

Determination of Median when frequency of any one class or classes are zero : When frequency of any class or classes are zero then we eliminate the such class or classes to find out the median. To eliminate such classes we add and deduct the half part of such class in preceeding and coming class-intervals respectively. After that we calculate the median on the basis of reconstructed series. If frequency of any class is zero but median-class do not affect with such class having the zero frequency, then there is no need to change the given frequency distribution and the said question can be solved as usual.

See the following illustration for clarification :

***Illustration* 27.**

Find out median from the following data :

Size :	0–10	10–20	20–30	30–40	40–50	50–60
Frequency :	5	7	13	0	18	7

Solution.

Size	f	c. f.
0–10	5	5
10–20	7	12
20–30	13	25
30–40	0	25
40–50	18	43
50–60	7	50

Median Group :

$$M = \text{Size of } \left(\frac{N}{2}\right) th \text{ item}$$

$$= \text{Size of } \left(\frac{50}{2}\right) th \text{ item}$$

$$\text{M} = \text{Size of } 25 \, th \text{ item}$$

There are two classes i.e. 20–30 and 30–40 for the said middle item of 25. Moreover frequency of 30–40 class is zero. Therefore, we will calculate the median after amending the class-interval having zero frequency. This class i.e. 30 – 40 will be adjusted in 20–30 and 40–50 classes. In this way amended class will be 20–35 and 35–50.

Amended distribution is as follows :

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Amended Median Class

Size	f	c. f.
0–10	5	5
10–20	7	12
20–35	13	25
35–50	18	43
50–60	7	50

Amended distribution is (20 – 35)

$$M = L_1 + \frac{i}{f}(m - c)$$

or $$= 20 + \frac{15}{13}(25 - 12)$$

or $$= 20 + \left(\frac{15}{13} \times 13\right) = 35$$

***Illustration* 28.**

Find out the value of median from the following frequency distribution :

Marks :	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45
f :	3	6	8	13	0	0	15	12	3

Solution.

Class 20–25 having zero frequency will be merged into 15–20 class and class 25–30 having zero frequency will be merged into 30–35 class. In this way amended frequency distribution will be as follows :

Marks :	0–5	5–10	10–15	15–25	25–35	35–40	40–45
f :	3	6	8	13	15	12	3
c. f. :	3	9	17	30	45	57	60

m = Size of $\left(\frac{N}{2}\right)$ *th* item or size of $\frac{60}{2} = 30$*th* item which lies in 30 *c.f.* and represent 15 – 25 as median class.

$$\text{Median } (M) = L_1 + \frac{i}{f}(m - c) \text{ or } 15 + \frac{10}{13} \times (30 - 17)$$

or $$= 15 + \left(\frac{10}{13} \times 13\right) \text{ or } 15 + 10 = 25 \qquad M = 25$$

***Illustration* 29.**

If the class-intervals of equal size have total frequencies of 390. First class-interval is 110-120. If the fourth, fifth, sixth, seventh and eighth class-intervals have cumulative frequencies of 190, 267, 327, 365 and 387 respectively, then what will be the median ?

Solution.

Computation of Median

Class	f	c.f.
110-120	–	–
120-130	–	–
130-140	–	–

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