

~~Q.~~ 16. In a group of 60 people, 40 speak Hindi, 20 speak both English and Hindi and all people speak at least one of the two languages. How many people speak only English and not Hindi? How many speak English?

**Sol.** Suppose  $H$  is the set of people speaking Hindi and  $E$  is the set of people speaking English. Then according to question,

$$n(E \cup H) = 60, n(H) = 40, n(H \cap E) = 20$$

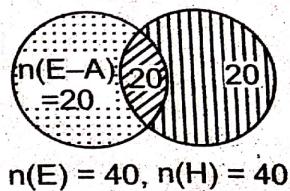
Using formula,  $n(E \cup H) = n(E) + n(H) - n(E \cap H)$

We get  $60 = n(E) + 40 - 20$

$\Rightarrow n(E) = 40$

Now,  $n(E - H) = n(E) - n(E \cap H)$

$$= 40 - 20 = 20$$



**Fig.**

**Ans.**

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12.

~~Q17.~~ In a town, 60% read magazine A, 25% do not read magazine A but read magazine B. Calculate the percentage of those who do not read any. Also find the highest and lowest possible figures of those who read magazine B.

**Sol.** Suppose total number is 100 and  $A$  be the set of those who read magazine A and  $B$  of those who read magazine B. Then according to problem.

$$n(A) = 60, n(A' \cap B) = 25$$

Obviously, sets  $A$  and  $A' \cap B$  are disjoint sets.

$$\text{Therefore, } A \cup B = A \cup (A' \cap B)$$

$$\Rightarrow n(A \cup B) = n(A) + n(A' \cap B) \\ = 60 + 25 = 85$$

$$\Rightarrow n(A \cup B)' = n(U) - n(A \cup B) \\ = 100 - 85 = 15$$

$\Rightarrow$  15% do not read any magazine.

Now,  $n(B)$  will be maximum when  $n(A)$  is minimum, i.e.,  $n(A) = 0$ .

$$\text{Then, } n(B) = n(A \cup B) = 85$$

$\Rightarrow$  Maximum number of  $B$  is 85%.

Similarly,  $n(B)$  will be minimum when  $n(A)$  is maximum.

$$\Rightarrow \text{Minimum } n(B) = n(A \cup B) - n(A) = 85 - 60 = 25$$

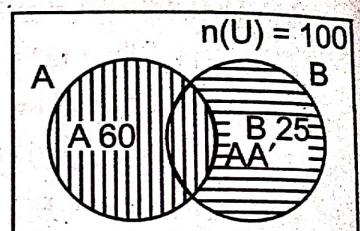


Fig.

Ans.

Theorem 2: State and prove De Morgan's laws.

or

To prove the following relations

(i)  $(A \cup B)' = A' \cap B'$

[B.C.A. (Meerut) 2009; B.C.A. (Avadh) 2006, 2009]

(ii)  $(A \cap B)' = A' \cup B'$

[B.C.A. (Meerut) 2002, 2008; B.C.A. (Lucknow) 2004, 2008]

Proof:

(i) Let  $x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A \text{ and } x \notin B$

$\Rightarrow x \in A' \text{ and } x \in B'$

$\Rightarrow x \in (A' \cap B')$

$\therefore x \in (A \cup B)' \Rightarrow x \in (A' \cap B')$

Then  $(A \cup B)' \subseteq A' \cap B'$  ... (1)

Again  $y \in A' \cap B' \Rightarrow y \in A' \text{ and } y \in B'$

$\Rightarrow y \notin A \text{ and } y \notin B$

$\Rightarrow y \notin A \cup B$

$\Rightarrow y \in (A \cup B)'$

$\therefore y \in A' \cap B' \Rightarrow y \in (A \cup B)'$

Then  $(A' \cap B') \subseteq (A \cup B)'$  ... (2)

From (1) and (2)  $(A \cup B)' = A' \cap B'$

(ii) Let  $x \in (A \cap B)' \Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A \text{ or } x \notin B$

$\Rightarrow x \in A' \text{ or } x \in B'$

$\Rightarrow x \in A' \cup B'$

$\therefore (A \cap B)' \subseteq A' \cup B'$  ... (1)

Again  $y \in A' \cup B' \Rightarrow y \in A' \text{ or } y \in B'$

$\Rightarrow y \notin A \text{ or } y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$

$\therefore A' \cup B' \subseteq (A \cap B)'$  ... (2)

From (1) and (2)

$(A \cap B)' = A' \cup B'$

**Theorem 3:** Use Distributive laws to prove the following:

(i)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(Union distributes intersection)

(ii)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(Intersection distributes union)

[B.C.A. (Meerut) 2001, 2006, 2007; B.C.A. (A.M.U.) 2010]

**Proof:**

(i) Let  $x \in A \cup (B \cap C) \Rightarrow x \in A$  or  $x \in (B \cap C)$

$$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C)$$

$$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$$

$$\Rightarrow x \in A \cup B \text{ and } x \in (A \cup C)$$

$$\Rightarrow x \in (A \cup B) \cap (A \cup C)$$

$$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \quad \dots(1)$$

Similarly it can be prove that

$$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \quad \dots(2)$$

From (1) and (2) we

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(ii) Let  $x \in A \cap (B \cup C) \Rightarrow x \in A$  and  $(x \in B \cup C)$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$

$$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$$

$$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$$

$$\Rightarrow x \in (A \cap B) \cup (A \cap C)$$

$$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \quad \dots(1)$$

Similarly it can be prove that

$$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \quad \dots(2)$$

From (1) and (2) we have

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Proved  
(2014, 15)

~~Q.23.~~ If  $A = \{1, 2, 4\}$ ,  $B = \{2, 5, 7\}$  and  $C = \{1, 3, 7\}$ , show that  $A \times (B \cap C) = (A \times B) \cap (A \times C)$ .

**Sol.** We have,  $A = \{1, 2, 4\}$ ,  $B = \{2, 5, 7\}$  and  $C = \{1, 3, 7\}$ .

$$B \cap C = \{7\}$$

$$\therefore A \times (B \cap C) = \{(1, 7), (2, 7), (4, 7)\}$$

Again,

$$A \times B = \{(1, 2), (1, 5), (1, 7), (2, 2), (2, 5), (2, 7), (4, 2), (4, 5), (4, 7)\}$$

and

$$A \times C = \{(1, 1), (1, 3), (1, 7), (2, 1), (2, 3), (2, 7), (4, 1), (4, 3), (4, 7)\}$$

$$\therefore (A \times B) \cap (A \times C) = \{(1, 7), (2, 7), (4, 7)\}$$

$$\text{Hence, } A \times (B \cap C) = (A \times B) \cap (A \times C)$$

Proved.

~~Q.24.~~ Let  $A = \{2, 3, 5\}$ ,  $B = \{3, 6, 8\}$  and  $C = \{4, 7, 9\}$ .

Show that  $A \times (B \cap C) = (A \times B) \cap (A \times C)$ .

**Sol.** We have,

$$A = \{2, 3, 5\}, B = \{3, 6, 8\} \text{ and } C = \{4, 7, 9\}$$

(2017)

$$B \cap C = \emptyset$$

$$\therefore A \times (B \cap C) = \emptyset$$

Again,

and

$$A \times B = \{(2, 3), (2, 6), (2, 8), (3, 3), (3, 6), (3, 8), (5, 3), (5, 6), (5, 8)\}$$

$$A \times C = \{(2, 4), (2, 7), (2, 9), (3, 4), (3, 7), (3, 9), (5, 4), (5, 7), (5, 9)\}$$

$$\therefore (A \times B) \cap (A \times C) = \emptyset$$

$$\text{Hence, } A \times (B \cap C) = (A \times B) \cap (A \times C)$$

Proved.

Q.25. If A and B are two sets such that  $A \cup B$  has 50 elements, A has 28 elements and B has 32 elements. How many elements does  $A \cap B$  have? (2016)

**Sol.** Given,  $n(A) = 28$ ,  $n(B) = 32$ ,  $n(A \cup B) = 50$

We know that  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$50 = 28 + 32 - n(A \cap B)$$

or

$$n(A \cap B) = 60 - 50 = 10$$

Hence,  $A \cap B$  has 10 elements.