Page-419

Que-33. Find the volume of ~~the~~ tetrahedron bounded by ~~the~~ the coordinate planes and the plane $x+y+z=1$.

Sol-.

$$\text{Volume} = \int_0^1\int_0^{1-x}\int_0^{1-x-y} dx\,dy\,dz$$

or

$$V = \int_0^1\int_0^{1-y}\int_0^{1-x-y} dy\,dx\,dz$$

or

$$V = \int_0^1\int_0^{1-z}\int_0^{1-x-y} dz\,dx\,dy$$

$x+y+z=1$.

Putting $y=0$ and $z=0$

$x=1$

$x+y+z=1$.

Putting $z=0$.

$x+y=1$

$y=1-x$

PAGE No

DATE : / 201

$$V = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx\, dy\, dz$$

$$V = \int_0^1 \int_0^{1-x} \left[ \int_0^{1-x-y} dz \right] dx\, dy$$

$$V = \int_0^1 \int_0^{1-x} \left[ z \right]_0^{1-x-y} dx\, dy$$

$$V = \int_0^1 \int_0^{1-x} \left[ 1-x-y-0 \right] dx\, dy$$

$$V = \int_0^1 \int_0^{1-x} (1-x-y)\, dx\, dy$$

$$V = \int_0^1 \left[ \int_0^{1-x} (1-x-y)\, dy \right] dx$$

PAGE No
DATE : / 201

$$V = \int_0^1 \left[ \int_0^{1-x} dy - \int_0^{1-x} x\, dy - \int_0^{1-x} y\, dy \right] dx$$

$$V = \int_0^1 \left[ [y]_0^{1-x} - x[y]_0^{1-x} - \left[\frac{y^2}{2}\right]_0^{1-x} \right] dx$$

$$V = \int_0^1 \left[ (1-x-0) - x(1-x-0) - \frac{1}{2}\left[(1-x)^2 - 0^2\right] \right] dx$$

$$V = \int_0^1 \left[ 1 - x - x + x^2 - \frac{1}{2}(1 + x^2 - 2x) \right] dx$$

$$V = \int_0^1 \left[ 1 - 2x + x^2 - \frac{1}{2} - \frac{1}{2}x^2 + \frac{1}{\cancel{2}} \times \cancel{2}x \right] dx$$

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$$V = \int_0^1 dx - \int_0^1 2x\,dx + \int_0^1 x^2\,dx$$

$$-\frac{1}{2}\int_0^1 dx - \frac{1}{2}\int_0^1 x^2\,dx$$

$$+ \int_0^1 x\,dx$$

$$V = [x]_0^1 - 2\left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x^3}{3}\right]_0^1$$

$$-\frac{1}{2}[x]_0^1 - \frac{1}{2}\left[\frac{x^3}{3}\right]_0^1$$

$$+\left[\frac{x^2}{2}\right]_0^1$$

$$V = (1-0) - \cancel{\frac{2}{2}}(1^2-0^2) + \frac{1}{3}(1^3-0^3)$$

$$-\frac{1}{2}(1-0) - \frac{1}{6}[1^3-0^3]$$

$$+\frac{1}{2}[1^2-0^2]$$

$$V = \cancel{1} - \cancel{1} + \frac{1}{3} - \cancel{\frac{1}{2}} - \frac{1}{6} + \cancel{\frac{1}{2}}$$

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