[R and S both are transitive]

$\Rightarrow \quad (a,c) \in R \cap S$

Therefore, $(R \cap S)$ is transitive on X.

From (1), (2) and (3), we have that $R \cap S$ is reflexive, symmetric and transitive and hence, $R \cap S$ is an equivalence relation on X. **Proved.**

Q.2. What do you understand by functions ? Describe the range and domain of a function with example.

Or **What do you mean by a function? Draw the graph of $f(x) = \frac{|x|}{x}, x \neq 0$.**

Functions

Ans.

Let A and B be two sets, then the rule or correspondence, which associates each element of A to a unique element of B, is called a function from set A to set B. If a general element of set A is denoted by x and of set B is denoted by y, then we say that y is a function of x, if for every $x \in A$, one and only one value of $y \in B$ can be determined.

Symbolically, If f is a function from a set A to a set B, then we write $f : A \rightarrow B$ read as f is a function from A to B or f maps A to B.

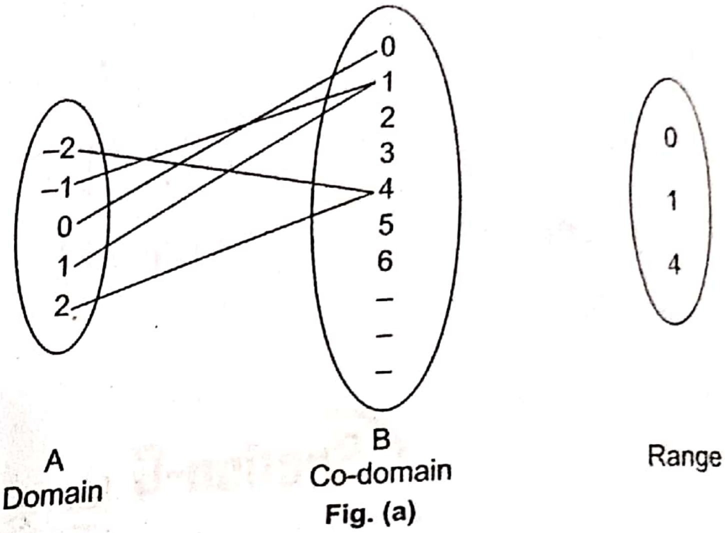

Fig. (a)

Range and Domain of a Function

Let an element $y \in B$ be corresponded by an element $x \in A$, then y is called the image of x and is denoted by $f(x)$, here, x is defined as the pre-image of y.

The set of all f-images of the element of A is called image set or the range of f and is denoted by

$$f(A) \text{ or } \{f(x) : x \in A\}$$

Evidently,

$$f(A) \subseteq B.$$

Thus, a mapping $f : A \rightarrow B$ is the set of ordered pairs $\{(a,b) : a \in A, b \in B\}$ so that no two ordered pairs have the same first element.

i.e. $f = \{(a,b) : a \in A, b \in B, b = f(x), \forall a \in A\}$

For example; Let $A = \{-2, -1, 0, 1, 2\}$ and B be the set of natural numbers for every $x \in A, f(x) \in B$ and $f(x) = x^2$.

Here, A is the domain and B is the co-domain, $f(a)$ is the value of the function $f(x)$, when x takes the value a *i.e.* when x is replaced by a. The elements of the co-domain which are equal to $f(x)$, form the range.

When $x = -2, f(-2) = (-2)^2 = 4$

When $x = -1, f(x) = 1$

When $x = 0, \quad f(x) = 0$

When $x = 1, \quad f(x) = 1$

When $x = 2, \quad f(x) = 4$

This can be illustrated in the adjacent figure:

If f is a function from set A to set B, then we write $f : A \rightarrow B$. The set A is the domain of f and the set B is the co-domain of f.

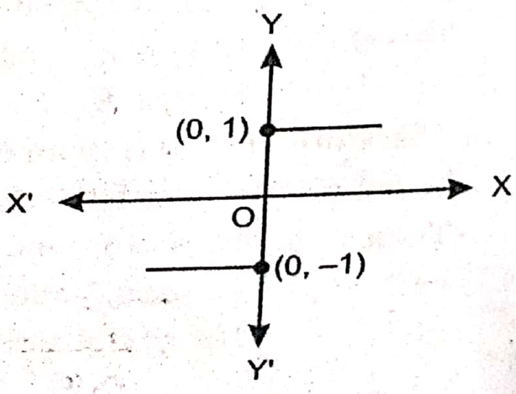

Fig. (b)

Graph of $f(x) = \frac{|x|}{x}$:

Given,

$$f(x) = \frac{|x|}{x}, x \neq 0$$

This function is known as Signum function.

Q.3. How many types of functions are there? Describe with example.

Ans.

Types of Functions

There are eight types of functions which are as follows :

1. One-one Function : A function f from A to B i.e. $f : A \to B$ is said to be one-one (or injective) iff distinct elements of A have distinct images.

Symbolically : f is one-one if for $x_1, x_2 \in A$, we have

$$x_1 \neq x_2$$

$\Rightarrow$ $$f(x_1) \neq f(x_2) \ \forall \ x_1, x_2 \in A$$

or $$f(x_1) = f(x_2)$$

$$x_1 = x_2 \ \forall \ x_1, x_2 \in A$$

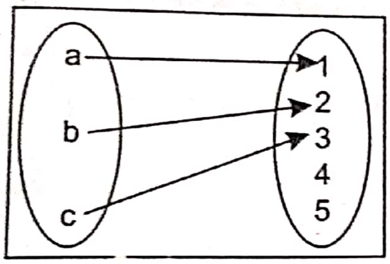

Fig. (a) One-one function.

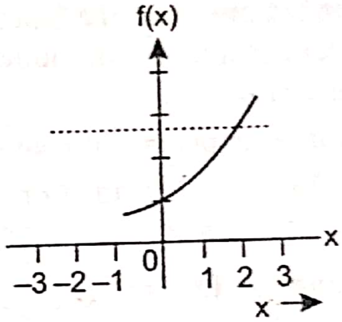

Fig. (b)

It is also called univalent function.

Graphically : A function is one-one if and only if no line parallel to x-axis meets the graph of the function in more than one point.

2. Many-one Function : A function $f : A \to B$ is called many-one if at least one element of co-domain B has two or more than two pre-images in domain A.

Symbolically : f is many-one if for $x_1, x_2 \in A$.

We can have $x_1 \neq x_2$

$\Rightarrow$ $$f(x_1) = f(x_2)$$

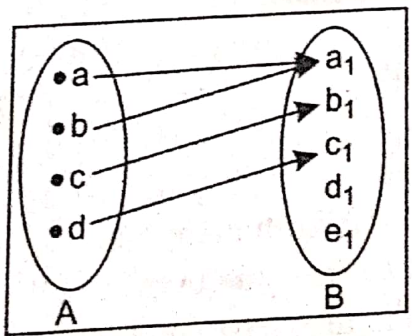

Fig. (c) Many-one function.

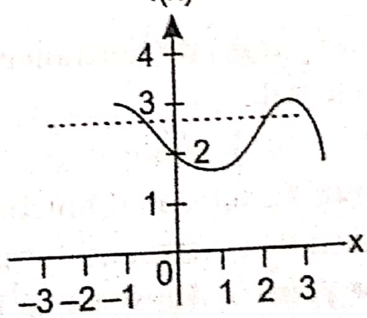

Fig. (d)

Complete

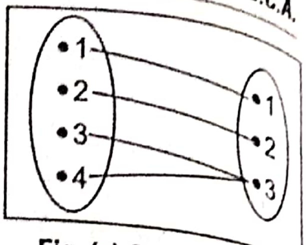

Fig. (e) Onto function.

Graphically : A function is many-one if and only if a line parallel to x-axis meets the graph of the function in more than one point.

3. Onto Function : A function $f : A \to B$ is called an onto function, if there is no element present in B which is not an image of some element of A, *i.e.* every element of B appears as the image of at least one element of A.

4. Into Function : A function $f : A \to B$ is called an into function if there is at least one element of the set B which has no pre-image in the set A.

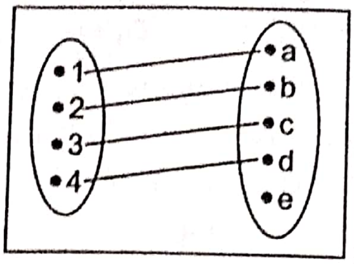

Fig. (f) Into function.

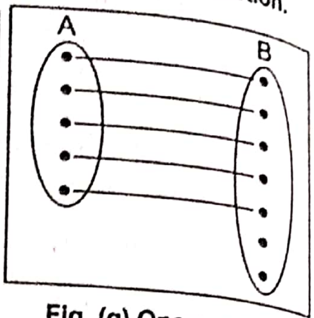

Fig. (g) One-one into function.

5. One-one Into Function : A function $f : A \to B$ is a one-one into function if in both one-one and into function *i.e.* the different points in A are joined to different points in B and there are some points in B which are not joined to any point in A.

Symbolically : One-one into function is defined as,

(a) Range $\subset$ co-domain. (b) $f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$.

6. One-one Onto Function : If a function $f : A \to B$ is both one-one and onto, *i.e.* the different points in A are joined to different points in B and no point in B is left vacant.

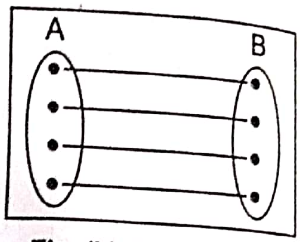

Fig. (h) One-one onto function.

Note : One-one onto mapping is also known as bijective.

Symbolically : One-one onto function is defined as,

(a) Range = Co-domain

(b) $x_1 \neq x_2$ and $f(x_1) \neq f(x_2)$

(c) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

7. Many-one Into Function : A function $f : A \to B$ which is both many one and into function is called a many one into function *i.e.*, two or more points in A are joined to some points in B and there are some points in B which are not joined to any point in A.

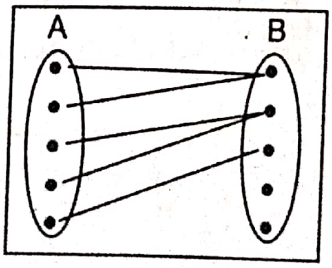

Fig. (i) Many-one into function.

Symbolically : Many-one into function is defined as,

(a) Range $\subset$ Co-domain

(b) $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$

8. Many-one Onto Function : If function $f : A \to B$ is both many one and onto function, then it is many one onto function, *i.e.* in B one point is joined to at least one point in A and two or more points in A are joined to some points in B.

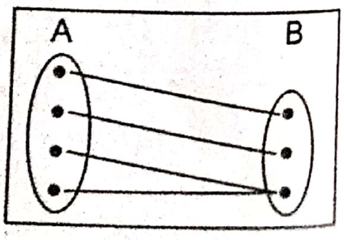

Fig. (j) Many-one onto function.