

$$Q1. \frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 8e^{3x}$$

$$Q2. (D^2 - D - 2)y = 8e^{2x} \sin 2x.$$

when P.I. $\frac{x^m}{x^m}$

Remember the following expansions —

$$(1-D)^{-1} = 1 + D + D^2 + D^3 + \dots \dots \dots \infty$$

$$(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \dots \dots \dots \infty$$

$$(1-D)^{-3} = 1 + 3D + 6D^2 + 10D^3 + \dots \dots \dots \infty$$

Note! — The terms will be alternatively +ve and -ve when the binomial is 1 + D.

$$Ex. \frac{d^2y}{dx^2} - 4y = x^3 \text{ or } (D^2 - 4)y = x^3$$

$$\text{S.o.l. C.P.} = C_1 e^{2x} + C_2 e^{-2x} \text{ or } (A \cosh 2x + B \sinh 2x)$$

$$P.I. = \frac{x^3}{D^2 - 4} = \frac{x^3}{4(1 - \frac{D^2}{4})} = \frac{1}{4} \left(1 - \frac{D^2}{4} \right)^{-1} x^3$$

$$= -\frac{1}{4} \left(1 + \frac{D^2}{16} + \frac{D^4}{16} + \dots \dots \right) x^3$$

$$= -\frac{1}{4} \left(x^3 + \frac{1}{4} x^5 \right) = -\frac{1}{8} x^3 (2x^2 + 3)$$

We need not have written $\frac{D^4}{16}$ as $\frac{1}{16} D^4 (x^3) = 0$

$$\therefore G.C. L.S. y = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{8} x^3 (2x^2 + 3).$$

$$Q1. (D^2 + 3D^2 + 2D)y = x^2$$

$$Q2. \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 4y = x^3$$

$$Q3. \frac{d^3y}{dx^3} + 3 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = x^2$$

$$\frac{\sqrt{16x^2 - 4x + 20}}{2}$$

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$$\text{Ex. } (\mathcal{D}^2 + \mathcal{D} + 1) Y = \sin 2x$$

C. P. is $e^{ix/2} (C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x)$

Want to find C. P. discuss on 1st April 2020.

$$\text{Now P.I. for } \sin 2x = \frac{\sin 2x}{\mathcal{D}^2 + \mathcal{D} + 1} = \frac{\sin 2x}{-(2)^2 + \mathcal{D} + 1} = \frac{\sin 2x}{\mathcal{D} - 3}$$

We have put $\mathcal{D}^2 = -a^2 = -(2)^2 = -4$

$$\text{or P.I.} = \frac{\mathcal{D} + 3}{\mathcal{D}^2 - 9} \sin 2x = \frac{\mathcal{D}(\sin 2x) + 3 \sin 2x}{-4 - 9}$$

$$= \frac{2 \cos 2x + 3 \sin 2x}{-13}$$

$$\text{G.S. is } Y = \text{C.P.} + \text{P.I.}$$

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$$\text{Sol. } \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + y = \alpha \cos 2x$$

$$\text{Sol. } A.E. : \mathcal{D}^2 - 4\mathcal{D} + 1 \text{ which C.P. is}$$

$$e^{2x} (C_1 e^{-\sqrt{3}x} + C_2 e^{\sqrt{3}x})$$

$$\text{P.I. for } \alpha \cos 2x = \frac{\alpha \cos 2x}{\mathcal{D}^2 - 4\mathcal{D} + 1} = \frac{\alpha \cos 2x}{-(2)^2 + \mathcal{D} + 1} = \frac{\alpha \cos 2x}{(4\mathcal{D} + 3)}$$

$$= -\alpha \frac{(4\mathcal{D} + 3)}{16\mathcal{D}^2 - 9} \cos 2x$$

$$= -\alpha \frac{4\mathcal{D}(\cos 2x) - 3 \cos 2x}{16(-4) - 9} = \frac{\alpha}{73} (-8 \sin 2x - 3 \cos 2x)$$

$$= -\frac{\alpha}{73} (8 \sin 2x + 3 \cos 2x).$$

So that complete solution (G.S.) is $y = \text{C.P.} + \text{I.F.}$

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(2)

Ex →

and the numerators shall be $(l \pm m)$ times
of multiplying l & m times and remembering
that means differentiation it becomes less and since

$$x^2(l-m) - m^2 = -a^2l + m^2$$

shall become constant.

Q. 4. $(D^2 - a^2)y = e^{3n}$
 Q. 5. $(D^2 - a^2 + b^2)y = e^{2n} - e^{-n}$ [Divide by e^{2n} and subtract]
 Q. 6. $(4D^2 + 4D + 3)y = e^{-2n}$

Our rule → Put in place of D^2 the quantity $-a^2$
 where x is of the form given at lesson.

Ans. $\frac{dy}{dx} = \frac{\text{Sum of } f(x)}{\text{Sum of } \cos ax} = \frac{f(-ax)}{(-a)^2}$

Solve the following equations —

$$Q1. (D^2 - 3D + 2)y = e^{5n}$$

$$Q2. (D^2 + 5D + 6)y = e^{-n}$$

$$Q3. (D^2 + 4D + 3)y = e^{2n}$$

$$Q4. (D^2 - a^2)y = e^{3n}$$

$$Q5. (D^2 - a^2 + b^2)y = e^{2n} - e^{-n}$$

$$Q6. (4D^2 + 4D + 3)y = e^{-2n}$$

$f(x) y = x$, where x is a function of x will consist of two parts —

- (i) complementary function (C.F.)
- (ii) Particular Integral (P.I.)

and then the General Solution is $y = C.F. + P.I.$, C.F. is obtained by writing the general solution of the equation $f(D)y = 0$, P.I. will be $\frac{x}{f(D)}$

Note :- Student should remember that D^1, D^2, D^3, \dots and so on stands for 1st, 2nd, 3rd diff. coeff. respectively whereas $\frac{1}{D}, \frac{1}{D^2}, \frac{1}{D^3}, \dots$ and so on for integrating one, twice, thrice and so on respectively.

* when x is of the form e^{ax} , where a is any const.

$$\text{P.I.} = \frac{x}{f(D)} = \frac{x}{f(a)}$$

our Rule is — if $f(D)$ put $x = a$ and P.I. will be calculated.

- i. P.I. = $\frac{e^{ax}}{f(D)}$ provided $f(a) \neq 0$ i.e. a is not a root of $f(D) = 0$.

$$\text{Ex. If } x = e^{2n} \text{ and } f(D) = D^2 + 3D + 5, \text{ then}$$

$$\text{P.I.} = \frac{e^{2n}}{D^2 + 3D + 5} = \frac{e^{2n}}{2^2 + 3 \cdot 2 + 5} = \frac{e^{2n}}{15} \quad \left[\begin{array}{l} \text{Put } x = 2 \\ \text{because } x = e^{2n} \\ \frac{x}{a} = e^{2n} \\ a = 2 \end{array} \right]$$

$$\text{If } x = e^{-\frac{n}{2}} \text{ and } f(D) = D^2 + D + 1 \text{ then}$$

$$\text{P.I.} = \frac{e^{-\frac{n}{2}}}{D^2 + D + 1} = \frac{e^{-\frac{n}{2}}}{(-\frac{1}{2})^2 + (-\frac{1}{2}) + 1} = \frac{e^{-\frac{n}{2}}}{\frac{1}{4}} = 4e^{-\frac{n}{2}}$$

Note :- In order to write the P.I. of $\sin nx$ or $\cos nx$ express them as $\frac{1}{2}(e^{inx} - e^{-inx})$ and $\frac{1}{2}(e^{inx} + e^{-inx})$ respectively.