J.S(P.G)COLLEGE SIKANDRABAD

M.COM-4TH SEMESTER

SUBJECT- OPERATIONS RESEARCH

TOPIC: Vogel's Approximation Method

Vogel's Approximation Method (VAM) or penalty method This method is preferred over the NWCM and VAM, because the initial basic feasible solution obtained by this method is either optimal solution or very nearer to the optimal solution.	
Vogel's Approximation Method (VAM) Steps (Rule)	
Step-1:	Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.
Step-2:	Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.
Step-3:	Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell. If there is a tie in the values of penalties then select the cell where maximum allocation can be possible
Step-4:	Adjust the supply & demand and cross out (strike out) the satisfied row or column.
Step-5:	Repeact this steps until all supply and demand values are 0.

Example-1

1. Find Solution using Voggel's Approximation method

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

SOLUTION:

TOTAL number of supply constraints : 3
TOTAL number of demand constraints : 4
Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

Table-1

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	10=40-30
S_3	40	8	70	20	18	12=20-8
Demand	5	8	7	14		
Column Penalty	21=40-19	22=30-8	10=50-40	10=20-10		

The maximum penalty, 22, occurs in column D_2.

The minimum c_{ij} in this column is c_{32} = 8.

The maximum allocation in this cell is min(18,8) = **8**.
It satisfy demand of D_2 and adjust the supply of S_3 from 18 to 10 (18 - 8 = 10).

Table-2

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20	10	20=40-20
Demand	5	0	7	14		
Column Penalty	21=40-19	--	10=50-40	10=20-10		

The maximum penalty, 21, occurs in column D_1.

The minimum c_{ij} in this column is c_{11} = 19.

The maximum allocation in this cell is min(7,5) = **5**.
It satisfy demand of D_1 and adjust the supply of S_1 from 7 to 2 (7 - 5 = 2).

Table-3

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20	10	50=70-20
Demand	0	0	7	14		
Column Penalty	--	--	10=50-40	10=20-10		

The maximum penalty, 50, occurs in row S_3.

The minimum c_{ij} in this row is c_{34} = 20.

The maximum allocation in this cell is min(10,14) = **10**.
It satisfy supply of S_3 and adjust the demand of D_4 from 14 to 4 (14 - 10 = 4).

Table-4

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	4		
Column Penalty	--	--	10=50-40	50=60-10		

The maximum penalty, 50, occurs in column D_4.

The minimum c_{ij} in this column is c_{14} = 10.

The maximum allocation in this cell is min(2,4) = **2**.
It satisfy supply of S_1 and adjust the demand of D_4 from 4 to 2 (4 - 2 = 2

Table-5

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10**(2)**	0	--
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	2		
Column Penalty	--	--	40	60		

The maximum penalty, 60, occurs in column D_4.

The minimum c_{ij} in this column is $c_{24} = 60$.

The maximum allocation in this cell is min(9,2) = **2**.
It satisfy demand of D_4 and adjust the supply of S_2 from 9 to 7 (9 - 2 = 7).

Table-6

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10**(2)**	0	--
S_2	70	30	40	60**(2)**	7	40
S_3	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	0		
Column Penalty	--	--	40	--		

The maximum penalty, 40, occurs in row S_2.

The minimum c_{ij} in this row is $c_{23} = 40$.

The maximum allocation in this cell is min(7,7) = **7**.
It satisfy supply of S_2 and demand of D_3.

Initial feasible solution is

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10**(2)**	7	9 | 9 | 40 | 40 | -- | -- |
S_2	70	30	40**(7)**	60**(2)**	9	10 | 20 | 20 | 20 | 20 | 40 |
S_3	40	8**(8)**	70	20**(10)**	18	12 | 20 | 50 | -- | -- | -- |
Demand	5	8	7	14		
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --		

The minimum total transportation cost =19×5+10×2+40×7+60×2+8×8+20×10=779

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6
∴ This solution is non-degenerate